Literacy Groups in the Math Classroom

Math Workshop Series

Implementing the Common Instructional Framework
Within a Mathematics Curriculum

JOBS FOR THE FUTURE

Developed by Robert Knittle
Mathematics Instructional Specialist
Literacy Groups in the Math Classroom

Developed by
Robert Knittle, Math Instructional Specialist
Jobs for the Future

Jobs for the Future is a nonprofit organization, engaged in research, policy analysis, consulting, and advocacy. We work to strengthen our society by creating educational and economic opportunity for those who need it most.
Literacy Groups in the Math Classroom

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Math Workshop Series

Implementing the Common Instructional Framework
Within a Mathematics Curriculum

**Training Session One**

Creating a Student-Centered Math Classroom

Algebraic Habits of Mind

Building Student Confidence: Translating Words to Numbers

Implementing the Common Instructional Framework to Develop College Readiness

Linear Inequalities and Absolute Value

Using Graphing Calculators with the Common Instructional Framework

**Training Session Two**

Systems of Equations and Inequalities

Creating Exponential Understanding

Rational Expressions and Matrices

Literacy Groups in the Math Classroom

Developing Logical Reasoning

Creating Formulas
University Park Campus School (UPCS) uses a common instructional framework consisting of six instructional strategies. These instructional strategies drive the instructional practice at UPCS and have led to its success. They also act as the core of the professional development program offered by the University Park Campus Institute, a partnership between UPCS and Jobs for the Future. These strategies create classrooms that allow for powerful learning and powerful teaching and form the basis of a coherent college-preparatory curriculum. They give all students of all skill levels access to the complex information needed to meet state and college-ready standards. These instructional strategies succeed because they engage all students in learning and require them to take an active role in their education.

**Collaborative Group Work:** Collaborative group work involves bringing students together in small groups for the common purpose of engaging in learning. Effective group work is well planned and strategic. Students are grouped intentionally with each student held accountable for contributing to the group work. Activities are designed so that students with diverse skill levels are supported as well as challenged by their peers. Collaborative group work uses questioning, scaffolding and classroom talk and centers literacy groups.

**Writing to Learn:** Writing to learn is a strategy through which students can develop their ideas, their critical thinking ability and their writing skills. Writing to learn enables students to experiment every day with written language and increase their fluency and mastery of written conventions. Writing to learn can also be used as formative assessment and as a way to scaffold mid- and high-stakes writing assignments and tests.

**Questioning:** Questioning challenges students and teachers to use good questions as a way to open conversations and further intellectual inquiry. Effective questioning (by the teacher and by students) deepens classroom conversations and the level of discourse students apply to their work. Teachers use this strategy to create opportunities for students to investigate and analyze their thinking as well as the thinking of their peers and the authors that they read in each of their classes.

**Scaffolding:** Scaffolding helps students to connect prior knowledge and experience with new information. Teachers use this strategy to connect students with previous learning in a content area as well as with previous learning in an earlier grade. Scaffolding also helps facilitate thinking about a text by asking students to draw on their subjective experience and prior learning to make connections to new materials and ideas.

**Classroom Talk:** Classroom talk creates the space for students to articulate their thinking and strengthen their voice. Classroom talk takes place in pairs, in collaborative group work and as a whole class. As students become accustomed to talking in class, the teacher serves as a facilitator to engage students in higher levels of discourse. Classroom talk opens the space for questioning, effective scaffolding and successful collaborative group work and literacy groups.

**Literacy Groups:** Literacy groups provide students with a collaborative structure for understanding a variety of texts and engaging in a higher level of discourse. Group roles traditionally drive literacy groups by giving each student a role to play and a defined purpose within the group. The specific roles or discussion guidelines may vary for different content areas, lengths of texts, or student level of sophistication using this strategy, but the purpose of literacy groups is to raise student engagement with texts by creating a structure within which they may do so.
Math Workshop Series
Literacy Groups in the Math Classroom

Guiding Questions

• How can teachers use literacy groups in the math classroom?
• What are the benefits of using literacy groups in the math classroom?
• What texts could teachers use in math literacy groups?

Materials

Literacy Group Roles
A Mathematical Perspective
Handmade Noodles/Doubling Grains of Rice
Ratio and Proportion text
Ratios, Proportions and the Geometric Mean text
chart paper and markers

Workshop Outcomes

By the end of this workshop, participants will

• recognize the importance of literacy groups as an instructional strategy
• understand how text-based discovery opens new avenues for learning
• understand how to implement literacy groups in a math classroom
• incorporate their prior knowledge and experience with the content of the workshop

Workshop Time: 90 minutes
Literacy Groups in the Math Classroom

Workshop Introduction
Time: 10 minutes

- Divide participants into groups of three as they walk into the room.
- Introduce the workshop and the guiding questions.
- Have participants write for 5 minutes to answer the following questions: What are literacy groups? How can they be used in the math classroom?
- Ask participants to share their responses. Define literacy groups using the Common Instructional Framework definition. Emphasize the difference between Collaborative Group Work and Literacy Groups.

Workshop Activity 1
Time: 50 minutes

- Have each participant read “A Mathematical Perspective.”
- Review the Literacy Group Roles and assign each participant a role.
- Using their assigned roles, have participants read Handmade Noodles or Doubling Rice Grains to generate a list of questions about the article from a mathematical perspective. Have each group write their questions on chart paper and post them around the room.
- Have participants do a Gallery Walk in their small groups. Upon returning to their tables, ask them to generate a list of three lesson extensions they could build based on the posted questions.
- Ask each small group to share their lesson extensions with the whole group.
- Ask participants other ways that literacy groups could be used in the math classroom and what some of the benefits might be in using literacy groups to learn math.

Workshop Activity 2
Time: 20 minutes

- Have participants get into teams of four.
- Ask them to create a lesson using the two texts on ratio and proportion that incorporates a literacy group to help students understand the concept of similarity. Have participants define the roles that students will play and determine the desired outcomes for the literacy group. Ask participants to clearly describe what students will know or produce as part of the literacy group on similarity.
- Have each team share their lesson with another team for feedback.
- Ask for volunteer teams to share their lessons with the whole group.

Workshop Closing
Time: 10 minutes

- Have participants write for 5 minutes to answer the following question: What are the benefits of using literacy groups in the math classroom?
- Have participants share their responses.
A Mathematical Perspective

As we investigate the world around us, it is vital that we use a mathematical perspective. Working together with our students to develop this perspective gives students a context for what they are learning in the math classroom and enables teachers and students to

• see patterns,
• find solutions to problems,
• uncover and interpret mathematical issues,
• connect to other disciplines, previous problems or future topics,
• make stories visual, and
• communicate findings about the context in which we live and the world around us.
Literacy Group Roles

Discussion Director
Develop a list of questions that your group will discuss about the reading. Direct the discussion by asking other members for their input based on their current role. Post the questions on chart paper. When you present, describe the work you accomplished as a group.

Connector
Your job is to connect the contents of the reading selection to at least two current or past real world events and experiences. You can also connect the reading to any school class you’ve taken. Be prepared to discuss the article with these connections in mind, helping the Director develop further questions your group might explore.

Engineer
Find several of the most important sentences and vocabulary words in the text. Underline what you consider the more important ones for discussion. Be prepared to discuss their impact on the overall article. See if your important words or sentences can help the Director develop further questions your group might explore.
**Handmade Noodles**

Chef Mark kneaded high-gluten white flour carefully along with the other ingredients of noodle dough in correct proportions: three cups of flour, half as much water, one-quarter teaspoon each of salt and baking soda. He vigorously swung and stretched the lump of dough out into a heavy single strand the length of his full two-arm span. Then he folded that long thick strand in half, and pulled the dough out again into its original length, so that two thinner strands now passed from one hand to the other.

Chef Mark: “Hello, everybody. I am the chef of the Dragon House in Wildwood, New Jersey. Today I will make noodles. Make the dough strong and smooth, keep the dough smooth and strong, and you will have noodles on the table.”

Fold one time: the dough becomes two noodles. Two times, and it becomes four noodles. Three, four times...ten, eleven, now twelve doublings, or four thousand and ninety-six noodles.

Wow! Almost five feet long, they are called dragon’s beard noodles—very fine, like a human hair. In two minutes, Chef Mark had drawn out four miles of fine noodles. (They were really two or three times as thick as human hair.) Legendary chefs of the past have gone to thirteen doublings, while experienced home noodle makers can complete eight or ten. But consider that if Chef Mark had continued the doubling, it would take only thirty-five more steps of doubling, six minutes’ work, before he would have reached what we know as atomic size. Of course, the actual procedure would fail long before that idealized atomic limit is reached.

The tantalizing nature of the doubling process is that the subdivision is so rapid. Some forty-six doublings would make noodles of true atomic fineness, in principle. But note that such an incredible feat would produce not a mere few miles of dragon’s beard, but noodles long enough to stretch to Pluto and beyond.

*Adapted with permission from*

http://raju.varghese.org/articles/powers2.html

Based on the book *The Ring of Truth*, by Philip and Phylis Morrison
Doubling Rice Grains

As told by Raju Varghese

The following story I heard as a kid while growing up in India. I do not know the original source of the legend nor if it is reproduced here as I heard it then. So consume it with an appropriate pinch of salt.

Once upon a time, long ago there was a king who ruled a prosperous land. Poverty was unknown there and every person was gainfully employed. Hence the sight of a beggar making his way along Main Street caused quite a stir in the capital of the land. The king demanded to see this strange man. When brought to him, the beggar revealed that he indeed did not have any possessions nor any money for the purchase of food. The king magnanimously offered all-you-can-eat meals for the rest of the week and clean clothes so that the beggar could continue his journey to the next land. Surprisingly, the beggar declined the royal offer and asked for a modest favor. The king demanded to know what the wish was. The beggar humbly requested a grain of rice for the first day, two on the second, four on the third day and so on--doubling the previous days contribution.

The king looked through the window at the overflowing granaries and almost accepted it when his grand vizier, remembering something that he had learned in Elements of Numbers (Math 201 at the local University) advised his highness that he should reconsider. To calculate the implication of the wish he pulled out a dusty abacus to perform mathematical calculations. He fumbled with it for a while but could not express the magnitude of the numbers involved because he ran out of beads. The king getting impatient with his vizier on such a simple wish from a poor man officially granted the beggar the wish. Little did he know that he had sounded the death knell of his reign.

The next day the beggar came to claim his grain of rice. The townsfolk laughed at the beggar and said that he should have taken the king's kind offer for a full meal instead of the measly grain of rice. On the second day he was back for the two grains. A week later, he brought a teaspoon for the 128 grains that was due to him. In two weeks it was a non-negligible amount of half a kilo. At the end of the month it had grown to a whopping 35 tons. A few days later the king had to declare bankruptcy. That is how long it was needed to bring down the kingdom.

For the pedantic ones, I assumed that a grain of rice weighs 0.033 grams. By the way, I used whole rice and hence your mileage might vary.

I have heard of a variation of this story from China where the beggar requested a grain for the first square on a chessboard, two for the second square and so on for each square. The king would be bankrupt long before the beggar got to the sixty-fourth square on the chessboard.
You solved problems by writing and solving equations.
You will solve problems by writing and solving proportions.
So you can estimate bird populations, as in Ex. 62.

If \( a \) and \( b \) are two numbers or quantities and \( b \neq 0 \), then the ratio of \( a \) to \( b \) is \( \frac{a}{b} \). The ratio of \( a \) to \( b \) can also be written as \( a : b \).

For example, the ratio of a side length in \( \triangle ABC \) to a side length in \( \triangle DEF \) can be written as \( \frac{2}{1} \) or 2:1.

Ratios are usually expressed in simplest form. Two ratios that have the same simplified form are called equivalent ratios. The ratios 7:14 and 1:2 in the example below are equivalent.

\[
\begin{align*}
\text{width of } RSTU &= \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2} \\
\text{length of } RSTU &= \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2}
\end{align*}
\]

**Example 1** Simplify ratios

**Simplify the ratio.**

a. \( 64 \text{ m} : 6 \text{ m} \)

b. \( \frac{5 \text{ ft}}{20 \text{ in.}} \)

**Solution**

a. Write \( 64 \text{ m} : 6 \text{ m} \) as \( \frac{64 \text{ m}}{6 \text{ m}} \). Then divide out the units and simplify.

\[
\frac{64 \text{ m}}{6 \text{ m}} = \frac{32}{3} = 32:3
\]

b. To simplify a ratio with unlike units, multiply by a conversion factor.

\[
\frac{5 \text{ ft}}{20 \text{ in.}} = \frac{5 \text{ ft}}{20 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{60}{3} = \frac{20}{1}
\]

**Guided Practice** for Example 1

Simplify the ratio.

1. 24 yards to 3 yards
2. 150 cm : 6 m
PROPORTIONS
An equation that states that two ratios are equal is called a proportion.

\[
\begin{align*}
\text{extreme} & \quad a = \frac{c}{d}, \quad \text{mean} \\
\text{mean} & \quad b = \frac{c}{d}, \quad \text{extreme}
\end{align*}
\]

The numbers \(b\) and \(c\) are the means of the proportion. The numbers \(a\) and \(d\) are the extremes of the proportion.

The property below can be used to solve proportions. To solve a proportion, you find the value of any variable in the proportion.

**KEY CONCEPT**

**A Property of Proportions**

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

   \[
   \frac{a}{b} = \frac{c}{d}, \quad \text{where} \quad b \neq 0 \quad \text{and} \quad d \neq 0, \quad \text{then} \quad ad = bc.
   \]

   \[
   \frac{2}{3} \cdot 6 = 3 \cdot 4 = 12
   \]

   \[
   2 \cdot 6 = 12
   \]

**EXAMPLE 4** Solve proportions

**ALGEBRA** Solve the proportion.

a. \(\frac{5}{10} = \frac{x}{16}\)

**Solution**

\[
\begin{align*}
5 \cdot 16 & = 10 \cdot x \\
80 & = 10x \\
x & = 8
\end{align*}
\]

(a) Write original proportion. Cross Products Property

\[
\begin{align*}
5 \cdot 16 & = 10 \cdot x \\
80 & = 10x \\
x & = 8
\end{align*}
\]

(b) Write original proportion. Cross Products Property

\[
\begin{align*}
1 \cdot 3y & = 2(y + 1) \\
3y & = 2y + 2 \\
y & = 2
\end{align*}
\]

**ANOTHER WAY**

in part (a), you could multiply each side by the denominator, 16.

Then \(16 \cdot \frac{5}{10} = 16 \cdot \frac{x}{16}\)

so \(8 = x\).

**GUIDED PRACTICE** for Example 4

Solve the proportion.

a. \(\frac{5}{x} = \frac{5}{8}\)

b. \(\frac{1}{y + 1} = \frac{2}{3y}\)

\[
\begin{align*}
5 \cdot 16 & = 10 \cdot x \\
80 & = 10x \\
x & = 8
\end{align*}
\]

\[
\begin{align*}
1 \cdot 3y & = 2(y + 1) \\
3y & = 2y + 2 \\
y & = 2
\end{align*}
\]

5. \(\frac{2}{x} = \frac{5}{8}\)

6. \(\frac{1}{x - 3} = \frac{4}{3x}\)

7. \(\frac{y - 3}{7} = \frac{y}{14}\)
6.2 Use Proportions to Solve Geometry Problems

You wrote and solved proportions.
You will use proportions to solve geometry problems.
So you can calculate building dimensions, as in Ex. 22.

Key Vocabulary
- scale drawing
- scale

In Lesson 6.1, you learned to use the Cross Products Property to write equations that are equivalent to a given proportion. Three more ways to do this are given by the properties below.

**KEY CONCEPT**

**Additional Properties of Proportions**

2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal.

\[
\frac{a}{b} = \frac{c}{d} \text{ then } \frac{b}{a} = \frac{d}{c}. 
\]

3. If you interchange the means of a proportion, then you form another true proportion.

\[
\frac{a}{b} = \frac{c}{d} \text{ then } \frac{a}{c} = \frac{b}{d}. 
\]

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

\[
\frac{a}{b} = \frac{c}{d} \text{ then } \frac{a + b}{b} = \frac{c + d}{d}. 
\]

**Example 1** Use properties of proportions

In the diagram \( \frac{MN}{RS} = \frac{NP}{ST} \).

Write four true proportions.

**Solution**

Because \( \frac{MN}{RS} = \frac{NP}{ST} \), then \( \frac{8}{10} = \frac{4}{x} \).

By the Reciprocal Property, the reciprocals are equal, so \( \frac{10}{8} = \frac{x}{4} \).

By Property 3, you can interchange the means, so \( \frac{8}{4} = \frac{10}{x} \).

By Property 4, you can add the denominators to the numerators, so

\[
\frac{8 + 10}{10} = \frac{4 + x}{x}, \text{ or } \frac{18}{10} = \frac{4 + x}{x}. 
\]
**Example 4** Use a scale drawing

**MAPS** The scale of the map at the right is 1 inch : 26 miles. Find the actual distance from Pocahontas to Algona.

**Solution**

Use a ruler. The distance from Pocahontas to Algona on the map is about 1.25 inches. Let x be the actual distance in miles.

\[
\frac{1.25 \text{ in}}{1 \text{ in}} = \frac{12.5 \text{ mi}}{26 \text{ mi}} \quad \text{actual distance}
\]

\[
x = 1.25(26) \quad \text{Cross Products Property}
\]

\[
x = 32.5 \quad \text{Simplify.}
\]

The actual distance from Pocahontas to Algona is about 32.5 miles.

**Example 5** Solve a multi-step problem

**SCALE MODEL** You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches.

a. What is the diameter of the actual dome?

b. About how many times as tall as your model is the actual building?

**Solution**

a. \[
\frac{10 \text{ in}}{560 \text{ ft}} = \frac{2.1 \text{ in}}{x \text{ ft}} \quad \text{measurement on model}
\]

\[
10x = 1176 \quad \text{Cross Products Property}
\]

\[
x = 117.6 \quad \text{Solve for } x.
\]

The diameter of the actual dome is about 118 feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

\[
\frac{560 \text{ ft}}{10 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 672
\]

The actual building is 672 times as tall as the model.

**Guided Practice** for Examples 4 and 5

4. Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map.

5. **What If?** Your friend has a model of the Reunion Tower that is 14 inches tall. What is the diameter of the dome on your friend’s model?
6.3 Use Similar Polygons

You used proportions to solve geometry problems.

You will use proportions to identify similar polygons.

So you can solve science problems, as in Ex. 34.

Key Vocabulary

- *similar polygons*
- *scale factor*

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, $ABCD$ is similar to $EFGH$. You can write "$ABCD$ is similar to $EFGH$" as $ABCD \sim EFGH$. Notice in the similarity statement that the corresponding vertices are listed in the same order.

**Corresponding angles**

$\angle A = \angle E$, $\angle B = \angle F$, $\angle C = \angle G$, and $\angle D = \angle H$

**Ratios of corresponding sides**

$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

**Example 1** Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a statement of proportionality.

**Solution**

a. $\angle R \equiv \angle X$, $\angle S \equiv \angle Y$, and $\angle T \equiv \angle Z$.

b. $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$, $\frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$, $\frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

c. Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

**Guided Practice** for Example 1

1. Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.
**SCALE FACTOR** If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the scale factor. In Example 1, the common ratio of $\frac{2}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

**EXAMPLE 2** Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of $\triangle ZYW$ to $\triangle FGHI$.

**Solution**

**STEP 1** Identify pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles $W$ and $I$ are right angles, so $\angle W \cong \angle I$. So, the corresponding angles are congruent.

**STEP 2** Show that corresponding side lengths are proportional.

$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4}, \quad \frac{XY}{GH} = \frac{30}{24} = \frac{5}{4}, \quad \frac{XW}{HI} = \frac{15}{12} = \frac{5}{4}, \quad \frac{WZ}{IF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

So $\triangle ZYW \sim \triangle FGHI$. The scale factor of $\triangle ZYW$ to $\triangle FGHI$ is $\frac{5}{4}$.

**EXAMPLE 3** Use similar polygons

**ALGEBRA** In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of $x$.

**Solution**

The triangles are similar, so the corresponding side lengths are proportional.

$$\frac{MN}{DE} = \frac{NP}{EF} \quad \text{Write proportion.}$$

$$\frac{12}{9} = \frac{20}{x} \quad \text{Substitute.}$$

$$12x = 180 \quad \text{Cross Products Property}$$

$$x = 15 \quad \text{Solve for } x.$$
Lesson 12.1
Ratio and Proportion

The amount a person uses his [or her] imagination is inversely proportional to the amount of punishment he [or she] will receive for using it.
— Anonymous

Similarity plays a significant role in human history. For example, accurate maps of regions of China have been found dating back to the second century B.C. The cartographers who created these maps must have used principles of similarity to be so accurate. Neolithic cave paintings are small-scale drawings of animals people hunted. Giant geoglyphs like the monkey shown at left, made by ancient Peru’s Nazca people (110 B.C.–800 A.D.), are some of the largest scale drawings ever created. Some of these animal figures measure over 400 feet long, and their shapes can be seen only from the air. Creating drawings of such scale was possible to do on the ground by using the principles of similarity you’ll learn about in this chapter.

The study of similar geometric figures involves ratios and proportions. You may be a little rusty working with ratios and proportions, so let’s review. What is a ratio?

A ratio is an expression that compares two quantities by division.

If $a$ and $b$ are two numbers, then the ratio of $a$ to $b$ is written “$a/b$.” The ratio of $a$ to $b$ is also written “$a:b$” or “$a$ is to $b$.”

**Example A**
Find the ratio of the shaded to the unshaded area.

![Shaded and Unshaded Areas](image)

\[
\frac{\text{Area}_{\text{shaded}}}{\text{Area}_{\text{unshaded}}} = \frac{6}{12} = \frac{1}{2}
\]

**Example B**
Find the ratio of $\bigcirc$ to $\bigstar$.

The ratio of $\bigcirc$ to $\bigstar$ is $\frac{3}{4}$.

When two ratios are equal, you have a proportion. The expression $\frac{3}{4} = \frac{6}{8}$ is a proportion.

A proportion is a statement of equality between two ratios.

Proportions are used to solve problems that involve comparing similar objects or situations. You may remember how to solve for a variable in an equation involving
fractions. If you have forgotten, one approach is to cross-multiply: if \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \). If one fraction is a multiple of the other, you may use a more direct method. Let’s look at a few examples.

**Example C**

If you work for two weeks and earn $380, how much will you earn in 15 weeks?

\[
\frac{380}{2} = \frac{x}{15} \\
2x = (380)(15) \\
x = \frac{(380)(15)}{2} \\
x = 2850
\]

You will earn $2850 in 15 weeks.

**Example D**

Solve for \( x \) in \( \frac{26}{50} = \frac{x}{75} \).

Before you cross-multiply, ask yourself, Can I reduce fractions? In this case, you can.

Rewrite \( \frac{26}{50} \) as \( \frac{13}{25} \). You then get the equation \( \frac{13}{25} = \frac{x}{75} \).

Next, before you cross-multiply, check to see if one numerator (or denominator) is a multiple of the numerator (or denominator) in the other fraction. In this problem, because \( 25 \cdot 3 = 75 \), \( x \) must be \( 13 \cdot 3 \), or 39.

Therefore \( x = 39 \).

**Exercise Set 12.1**

1. Find the ratio of \( \frac{3}{5} \) to \( \frac{2}{3} \).

2. *Find the ratio of the shaded area to the area of the whole figure.*

3. Use the figure below to find these ratios: \( AC/CD \), \( CD/BD \), and \( BD/BC \).

4. Find the ratio of the perimeter of triangle \( RSH \) to the perimeter of triangle \( MFL \).

5. Find the ratio of the area of triangle \( RSH \) to the area of triangle \( MFL \).
Lesson 12.2

Similarity

He that lets the small things bind him
Leaves the great undone behind him.
— Piet Hein

Similarity plays an important part in the construction of such large objects as cars and trucks and such small objects as integrated circuits. Before building a car, engineers design scale drawings of the car, use the scale drawings to build scale models, then run tests with the scale models. To fabricate integrated circuits, electrical engineers use a computer to create a large-scale map of the integrated circuit. They then reduce the circuit design and transfer it onto minute silicon chips.

Movies are comprised of scenes scaled down to small images on a piece of film. These images are then scaled way up again to a large screen. Similarity is used not only in the movie-making process, but also as a theme of many movies. The movie King Kong was about a giant 30-foot gorilla similar in shape to real gorillas. In the movie Fantastic Voyage, people in a submarine were shrunk proportionally until they were small enough to be injected into the bloodstream of another person. Shrinking people may be a far-fetched idea, but current research in nanotechnology is making the possibility of tiny machines taking such a voyage more real than fantastic.

Figures that have the same shape and the same size are congruent figures. Figures that have the same shape but not necessarily the same size are similar figures. This, however, is not a precise definition for similarity. What does it mean to be the same shape?

A person looking at reflections in different fun-house mirrors sees different images. Do we want to say that the images are similar to the original? They certainly have a lot of features in common, but they are not similar in a mathematical sense. Similar shapes can...
be thought of as enlargements or reductions with no irregular distortions. If you can place figure A on a photocopy machine and enlarge or reduce it to fit perfectly over figure B, then figure B and the original figure A are similar. Are all rectangles similar? They certainly have many common characteristics, but they are not all similar because some cannot be enlarged or reduced to fit perfectly over others. What about other geometric figures? All squares are similar to one another and all circles are similar to one another, but all triangles are not similar to one another. In this lesson you will arrive at a mathematical definition for similar polygons.

The two illustrations in Investigation 12.2.1 represent the enlargement and the reduction of an impossible three-dimensional solid.

Let's use the enlarged and reduced figures to see what makes polygons similar. Even though the impossible figures appear to be three-dimensional, please try to see each figure as just a two-dimensional figure.

**Investigation 12.2.1**

Use patty papers to compare all corresponding angles. Measure the corresponding segments in both hexagons. Find the given ratios of the lengths of corresponding sides.

Is \( \angle ABC \equiv \angle PQR \)?

Is \( \angle BCD \equiv \angle QRS \)?

How do the other corresponding angles compare?

<table>
<thead>
<tr>
<th></th>
<th>( AB )</th>
<th>( BC )</th>
<th>( CD )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AB )</td>
<td>( ? )</td>
<td>( ? )</td>
<td>( ? )</td>
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<tr>
<td>( BC )</td>
<td>( ? )</td>
<td>( ? )</td>
<td>( ? )</td>
</tr>
<tr>
<td>( CD )</td>
<td>( ? )</td>
<td>( ? )</td>
<td>( ? )</td>
</tr>
</tbody>
</table>

How do the corresponding sides of similar polygons compare? From your observations, you should be able to state that if two polygons are similar, then their corresponding angles are congruent and their corresponding sides are proportional. This statement is reversible. That is, if you construct two polygons that have corresponding angles congruent and corresponding sides proportional, then one polygon is an enlargement or a reduction of the other. In other words, one polygon is similar to the other. Based on these observations, let's state a more mathematical definition for similar polygons.

Two polygons are *similar polygons* if and only if the corresponding angles are congruent and the corresponding sides are proportional.
The symbol for the words is similar to is \( \sim \). You use this symbol in the same way you use the symbol for congruence. If the two quadrilaterals CORN and MAIZ are similar, you write "CORN \( \sim \) MAIZ." Just as in statements of congruence, the order of the letters tells you which segments and which angles in the two polygons correspond.

\[
\begin{align*}
\text{CORN} & \sim \text{MAIZ} \\
\frac{CO}{MA} & = \frac{OR}{AI} = \frac{RN}{ZN} = \frac{NC}{ZM} \\
\angle C & \equiv \angle M \\
\angle O & \equiv \angle A \\
\angle R & \equiv \angle I \\
\angle N & \equiv \angle Z
\end{align*}
\]

In this investigation you discovered that two polygons are similar if and only if their corresponding angles are congruent and the corresponding sides are proportional. Do we need both conditions to guarantee that the two polygons are similar? In other words, if you know only that the corresponding angles of two polygons are congruent, can you conclude that the polygons have to be similar? Or, if corresponding sides of two polygons are proportional, are the polygons necessarily similar?

**Polygons with corresponding angles congruent**

\[
\begin{align*}
\angle S & \equiv \angle R \\
\angle Q & \equiv \angle E \\
\angle U & \equiv \angle C \\
\angle A & \equiv \angle T
\end{align*}
\]

However, \( \frac{12}{10} \neq \frac{12}{18} \).

Corresponding angles of square SQUA and rectangle RECT are congruent, but their corresponding sides are not proportional. Clearly the two polygons are not similar. Therefore you cannot determine whether or not two polygons are similar based only on the fact that the corresponding angles of the two polygons are congruent.

**Polygons with corresponding sides proportional**

\[
\begin{align*}
\frac{12}{18} & = \frac{12}{18} \\
\text{However, } \angle S & \neq \angle R.
\end{align*}
\]

Corresponding sides of square SQUA and rhombus RHOM are proportional, but their corresponding angles are not congruent. Clearly the two polygons are not similar. Therefore you cannot determine whether or not two polygons are similar based only on the fact that the corresponding sides of the two polygons are proportional.

You have just discovered from the two counterexamples above that to determine
whether or not two polygons are similar, you must know both that the corresponding sides of the two polygons are proportional and that the corresponding angles are congruent.

You can use the definition of similar polygons to find missing measures in similar polygons.

**Example A**

Find the measure of the side labeled \(x\) and the measure of the angle labeled \(y\) in the similar polygons below.

\[
\text{SMAL} \sim \text{BIGE}
\]

\[
\frac{18}{24} = \frac{21}{x} \\
\frac{3}{4} = \frac{21}{x} \\
3x = (4)(21) \\
x = 28
\]

So the measure of the side labeled \(x\) is 28°.

\[
\angle M \cong \angle I
\]

So the measure of the angle labeled \(y\) is 83°.

You can also use the definition of similar polygons to determine whether two polygons are similar if you know the measures of their angles and the lengths of their sides. Look at Example B.

**Example B**

Determine whether or not the polygons below are similar.

\[
\frac{15}{25} = \frac{5}{9} \quad \text{and} \quad \frac{25}{45} = \frac{5}{9},
\]

but \(\frac{20}{40} = \frac{1}{2}\).

Therefore the triangles are not similar.

An image on movie film and the image projected onto the screen are similar (as long as the movie is projected at right angles to the screen). For best projection results, a projector is placed a fixed distance away from a screen. If the projector is moved half this distance to the screen, each dimension of the usual image is cut in half. If the projector is moved three times this fixed distance away from the screen, the dimensions of the new image are all three times its usual size.
Additional Resources
Literacy Group on Similarity

After reading the text with your role in mind, talk to your group members to help create a poster that shows the key ideas about similarity. Your poster should be split into four parts to represent the findings of each role within your group.

Literacy Group Roles

Strategizer – Document the ways that your group found to understand the reading. What strategies did you and your group members use to help create an understanding of the text? Were you underlining, creating a visual image or thinking back to previous topics? Were there other ways your group strategically made sense of the text? Check in with your group members to document the strategies they used to understand the reading.

Energizer – Find the most important sentences in the text and underline them. Make a list of vocabulary that you aren’t sure about and need to define or words that are important and will help describe the text. Check in with your group members to see what they found important in the reading. Document a list of the main points. Help keep your group energized and on task.

Synthesizer – Make connections between the text and other content areas. Help your group generate at least two examples of how you might apply this knowledge in the real world.

Organizer – Summarize the discussion and prepare your poster. The findings of each role should be displayed on the poster (listing of strategies, listing of important sentences and vocabulary, listing of main ideas and applications to the real world). Be prepared to represent your group and explain your poster. Seek advice from your group members and engage them in the process of creating the poster.
40 YEARS FROM NOW
Teaching Guidelines

Subject: Mathematics
Topics: Algebra--Patterns, Functions and Relations; Linear Equations, Quadratic Relations and Functions, Exponential Functions
Grades: 7 – 9

Concepts:
• Function

Knowledge and Skills:
• Can extrapolate a graph when a pattern exists
• Can plot a point in a two-dimensional coordinate system, given the coordinates, or determine the coordinates of a given point
• Can determine the equation of a linear function that closely matches a set of points
• Can determine a quadratic function that closely matches a set of points
• Can determine an exponential function that closely matches a set of points

Materials: None

Procedure: This project should be done by students individually or in teams of two.
Distribute the handout and discuss it. Ensure that students understand the assignment.
You may wish to leave it to the students to work out a technique for predict future values, or work out the procedure in class discussion and have the students apply it.
For less advanced students, a good technique is to simply graph the given data (making sure the horizontal axis extends from 1956 to around 2050) and then extend the curve visually.
More advanced students should be instructed to come up with a mathematical model* that most closely describes the given data, and use that to predict the value 40 years into the future.

*That is, a function chosen from among the ones the students have experience with: linear, quadratic, exponential, logarithmic, polynomial, or sinusoidal.
E-mail printout  10:48 am

To: Research team
From: Features editor
Subject: 40 years from now

A lot of magazines have a feature like “40 years ago” where you can find out about things that happened in the past. But I want to do something a little different: “40 years from now.”

Please look at the data below, which gives Olympic winning times for various sports for the last 40 years or so, and work out what you think the times will be 40 years from now.

You can pick two out of the four sports.

I don’t just want your opinion on this— I need something that is mathematically accurate and I need an explanation of how you got your numbers so we can explain it to our readers. Your explanation should include diagrams or graphs if possible—readers like pictures.

By the way, I expect you will see some interesting patterns in these numbers. Let me know what they are because we may want to do a separate story on that.

Janet

<table>
<thead>
<tr>
<th></th>
<th>Women’s 500 meter speed skating (seconds)</th>
<th>Men’s 200 meter freestyle swimming (seconds)</th>
<th>Men’s, 400 meter run (seconds)</th>
<th>Women’s long jump (meters)</th>
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<td>n/a</td>
<td>46.7</td>
<td>6.35</td>
</tr>
<tr>
<td>1960</td>
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<td>44.9</td>
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<tr>
<td>1964</td>
<td>45.0</td>
<td>n/a</td>
<td>45.1</td>
<td>6.76</td>
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<tr>
<td>1968</td>
<td>46.1</td>
<td>1:55.2</td>
<td>43.8</td>
<td>6.82</td>
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<td>43.33</td>
<td>1:52.78</td>
<td>44.66</td>
<td>6.78</td>
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<tr>
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<td>42.76</td>
<td>1:50.29</td>
<td>44.26</td>
<td>6.72</td>
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<tr>
<td>1980</td>
<td>41.78</td>
<td>1:49.81</td>
<td>44.6</td>
<td>7.06</td>
</tr>
<tr>
<td>1984</td>
<td>41.02</td>
<td>1:47.44</td>
<td>44.27</td>
<td>6.95</td>
</tr>
<tr>
<td>1988</td>
<td>39.10</td>
<td>1:47.25</td>
<td>43.87</td>
<td>7.40</td>
</tr>
<tr>
<td>1992</td>
<td>40.33</td>
<td>1:46.70</td>
<td>43.5</td>
<td>7.14</td>
</tr>
<tr>
<td>1996</td>
<td>39.28*</td>
<td>1:47.63</td>
<td>43.49</td>
<td>7.12</td>
</tr>
</tbody>
</table>

*The final speed skating figure is for the year 1994, not 1996.
World Population Study

Grade Level: 7-12
This lesson is designed for students in grade levels 7 to 12 who have mastered basic math concepts or can use a calculator to solve basic operations. This lesson is a cross-curricular design.

Overview: The concept of exponential (vs. linear relationships) is a difficult concept for many students to understand. This lesson helps students understand the difference between the two and relates this knowledge to human population growth over time.

Purpose: The purpose of this lesson is to help students learn about an exponential relationship and how it relates to human population growth and the current global population crisis. Students will learn how to graph both exponential and linear information.

Materials: Graph paper, pencils, rulers, calculators, blackboard & chalk.

Objectives: Students will learn how to:
1. Solve a real-life math problem involving multiple and sequential steps in order to answer a question.
2. Graph the results of their problem solving to give a visual representation of the results.
3. Explain the difference between a linear and an exponential relationship.
4. Apply this knowledge to a study of world population growth by making a graph of world population data from 1650 to 2050 (projected).
5. Explain some of the reasons for the growth in the world’s population.

To understand why world population is now growing so fast, we will discuss some issues. This activity will help you understand one of them. Read the four “family histories” below and answer the questions. It might be useful to draw a “family tree” for each one to help you with the math.

Family A: Family A has one child. If that child has one child, how many grandchildren does Family A have? If the grandchild has one child, how many great grandchildren does Family A have?

Family B: Family B has two children and each of them has two children. How many grandchildren does Family B have? If each grandchild has two children, how many great-grandchildren does Family B have?

Family C: Family C has three children and each of them has three children. How many grandchildren does Family C have? If each grandchild has three children, how many great-grandchildren does Family C have?

Family D: Family D has four children and each of them has four children. How many grandchildren does Family D have? If each grandchild has four children, how many great-grandchildren does Family D have?
**Tying It All Together**
The number of children multiplies each generation. For Family B there are twice as many children each generation and for Family D there are four times as many. Few families really have the same number of children each generation. But these examples help explain one reason why the world’s population has grown rapidly in the last 100 years.

Another reason is that in most areas of the world, people are living longer. Up until 125 years ago, the world’s population was increasing slowly. Although the number of births multiplied, many babies did not live and large numbers of children and adults died from diseases. Over the past 150 years, diet, nutrition, and health care have improved. Scientists have discovered cures for many diseases. As a result, the death rate has been declining rapidly. With more people being born and living longer, the result has been a big jump in the world’s population.

There are concerns that as world population increases there will be shortages of food, water, and the quality of life will be threatened worldwide. What do you think?

*Adapted from Margaret V. Smith, Reg. II Observation & Assessment Center, Salt Lake City, UT*
Mathematical Literacy

By Hope Martin

What is mathematical literacy? To answer this question we must look at the meaning of literacy in general. In the narrowest sense, literacy refers to the ability to read, write, speak, and use language. Literacy is “not in isolated bits of knowledge but in students’ growing ability to use language and literacy in more and broader activities” (Moll, 1994, p. 202).
Just as knowing the definitions of words does not make a person literate, knowing rules and algorithms to solve mathematics problems does not make a person mathematically literate. Mathematical literacy implies that a person is able to reason, analyze, formulate, and solve problems in a real-world setting. Mathematically literate individuals are informed citizens and intelligent consumers. They have the ability to interpret and analyze the vast amount of information they are inundated with daily in newspapers, on television, and on the Internet.

**Mathematical Literacy in the Classroom**

The traditional mathematics curriculum is linear and based on a scope and sequence of skills and algorithms. Steen (1990) observed that traditional mathematics has chosen a few strands (e.g., arithmetic, algebra, and geometry) and arranged them horizontally to form a curriculum. But these courses give a distorted view of mathematics as a science and do not relate to either the educational experiences of students or the reality of their world. Ellis (2001) discovered that students leaving an elementary algebra class could solve fewer real-world problems after they completed the course than before they took it. Before they studied algebra, students used reasoning, previous knowledge, and arithmetic. After taking algebra, many believed that the only way to solve the problems was by using variables and setting up equations. There has been research regarding not the content of mathematics but the structure of the system—moving from a scope-and-sequence model to a model that relates to the needs of students.

The Organisation of Economic Co-operation and Development (OECD, 2002) claims that teaching students to “mathematize” should be a primary goal of mathematics education. Mathematizing involves five elements:

1. Starting with a problem whose roots are situated in reality
2. Organizing the information and data according to mathematical concepts
3. Transforming a real-world, concrete application to an abstract problem whose roots are situated in mathematics
4. Solving the mathematical problem
5. Reflecting back from the mathematical solution to the real-world situation to determine whether the answer makes sense.

I was a teacher for more than 30 years, and it is the last of the five elements of mathematizing that made me question the way mathematics is currently taught. I remember asking a student in one of my seventh-grade classes if the answer she got to a division of fraction problem made any sense. Without blinking an eye she stated, “Math hasn’t made any sense since fifth grade. The rule for dividing fractions doesn’t make sense!” Believe it or not, her remark took me back many years to Mrs. Sullivan’s fifth-grade classroom in P.S. 79. I remember Mrs. Sullivan saying, “Just change the division sign to a multiplication sign and turn over the second number.” When I ask why we were supposed to do this, she said, "Because that’s the rule!" At the time, I probably thought that math didn’t make much sense as well.

Before children enter school, they view mathematics as a useful way to quantify and understand their world. But when they enter a mathematics classroom where the primary
focus is on the memorization of facts and rules and there is a preoccupation with such low-level computation skills as defining, memorizing, recalling, repeating, and explaining, students lose their belief that math is a sense-making experience. School math and real math become disconnected and mathematics is no longer a tool that humans use to interpret and quantify their world, but something they learn to pass the class and move on to the next grade.

Another current theory takes a similar view regarding the need to reconstruct mathematics education. Realistic Mathematics Education (RME) maintains that all human activity must be connected to reality, including mathematics (Freudenthal, 1991). This theory has five components:
1. Using a real-world context as a starting point for learning
2. Bridging the gap between abstract and applied mathematics by using visual models
3. Having students develop their own problem-solving strategies rather than memorize rules and procedures
4. Making mathematical communication, perhaps in the form of journaling or oral presentations, an integral part of the lesson
5. Making connections to other disciplines using meaningful real-world problems.

It appears that innumeracy or mathematical illiteracy may not be the result of the content taught, but the pedagogy used to teach it. Perhaps traditional mathematics instruction is too formal, less intuitive, more abstract, less contextual, more symbolic, and less concrete than the type of instruction that would expand student thinking and develop mathematical literacy. Current thinking favors starting with a problem whose roots are situated in the reality of the student.

In the real world, phenomena that lend themselves to mathematical solutions do not come organized in a neat little package the way textbook problems do. And rarely can problems taken from the real world be solved in only one way with a prescribed strategy. The skills and concepts needed to solve these problems involve multiple strands of mathematics, and students must use different strategies to solve them. It also appears that the linear scope-and-sequence approach most commonly used to teach mathematics may work in opposition to developing mathematical literacy.

An Integrated Whole
In the 1980s, there were great concerns about mathematical illiteracy or innumeracy in the United States. The National Research Council (NRC, 1989) identified mathematics as the key to opportunity and regarded mathematics as “a filter, rather than a pump” because the lack of an adequate background in mathematics severely limited career opportunities (p. 6). In response to these concerns, the National Council of Teachers of Mathematics (NCTM) developed Curriculum and Evaluation Standards for School Mathematics, the first comprehensive set of mathematics standards. This document was updated and revised in 2000 in Principles and Standards of School Mathematics.

NCTM divided 10 standards in two groups: the Content Standards and the Process Standards. The five Content Standards—Number and Operation, Algebra, Geometry, Measurement, and Data Analysis and Probability—describe the content students should learn. The five Process Standards—Problem Solving, Reasoning and Proof, Communication, Connections, and Representation—describe what students should be doing to acquire and use the content knowledge. These standards do not reflect the traditional mathematics curriculum: “This set of ten Standards does not neatly separate the school mathematics curriculum into nonintersecting sets.... Process can be learned with the Content Standards and content can be learned within the Process Standards. Rich connections and intersections abound” (NCTM, 2000, pp. 30–31).

Although NCTM chose to identify strands of mathematics in a more traditional framework, the Programme for International Student Assessment group at OECD (2002) created four phenomenological categories they believe better describe the makeup of mathematics: quantity, space and shape, change and relationships, and uncertainty. NCTM’s current mathematical strands can fit into these categories.

Quantity. As people explore the world, they use such terms as tall or short, more or less, many or few. This category includes measurement; numerical patterns; number sense; examining different representations of numbers; understanding the meaning of operations, estimation, and mental arithmetic; understanding the magnitude of numbers; and being able to communicate mathematically.

Space and shape. Although this category has ties to traditional geometry, it goes far beyond it. In mathematics, geometric patterns serve as relatively simple examples of the more-complex patterns of maps, architecture, and art. In students’ studies of geometric shapes, they learn to identify and name different shapes, explore similarities and differences, and study their physical dimensions. Both two- and three-dimensional objects are included in this category of study.
Change and relationship. All natural phenomena show change: organisms change as they grow, the fluctuations in the stock markets of the world reflect change, the weather changes, and the balance in bank accounts changes on the basis of the size and frequency of deposits and withdrawals. Mathematical relationships take the form of equations or inequalities or the effect that a change in one variable has on another. For example, direct variation shows that an increase in the first variable causes an increase in the second (e.g., as car speeds increase so does the chance of a fatal accident). Indirect variation shows that an increase in the first variable causes a decrease in the second variable (e.g., the time it takes to reach a destination decreases with an increase in the speed of the car).

Uncertainty. The ability to deal with uncertainty should be part of a strong curriculum that deals with data and chance. Data are not merely numbers, but numbers in context. They relate to information that has been collected and to have meaning they must be carefully analyzed—how was it collected? is it relevant? does it inform? is it accurate? The important mathematical concepts in this category are the collection, analysis, and graphic display of data and probability.

Integration
Suppose the topic for the day in the math classroom is place value and the study of large numbers. It can, of course, be taught using traditional textbook materials and place value charts. Is it possible to teach this topic using strategies that give students the opportunity to make real-world connections and encourage them to compare numbers of enormous magnitude to something with which they are familiar so the lesson takes on meaning? Suppose the teacher asked this question: In August 2006, the national debt was recorded on www.brillig.com/debt_clock at $8,507,066,972,861.55. If every man, woman, and child in the United States had to pay a fair share of the national debt, how much would that be? What is a fair share of the national debt?

Assuming a population of 300 million, the answer to this question is that each person’s share is $28,356.89. To make this number more meaningful, students can do some research on www.census.gov and learn that this number is very close to the median income of workers 15 years old and older in the United States. As a follow-up, suppose students were given the opportunity to investigate other large numbers and prepare a presentation relating them to more understandable numbers, such as how long 1 million seconds or 1 billion seconds is in more appropriate units. As with all investigations that begin with real-world applications, this one spans a number of mathematics skills and concepts, such as numbers and numeration, data collection and statistics, the use of technology, and mathematical communication.

Motivation to Learn
“Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for higher-order thinking, and for versatile problem-solving” (NRC, 1989, p. 57). In the current climate of high-stakes testing, it sometimes seems necessary to revert to traditional strategies to teach mathematics skills. These are meeting with limited success—the students who are at the most risk remain at risk.

It is important to engage students in thought-provoking, meaningful mathematics. Mathematics should be taught using strategies that encourage mathematical literacy because when students ask, “When are we ever going to use this?” they are telling their teachers that they do not see the relevancy and importance of what they are being taught. When real-world applications are used in the mathematics classroom, student interest is piqued and they are motivated to learn.

References
Writing and Mathematics: An Exponential Combination

Traditional academic disciplines in high schools often resemble silos. The grain stored in one never interacts with the grain stored in another. The discrete disciplines that frequently define the day of high school students resembles no known line of work other than teaching in high schools themselves. Once out of high school, students must quickly adapt to a world where the boundaries that defined their high school experience barely exist.

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They are the coauthors of Write for Mathematics (2006, Corwin).

PREVIEW

The academic silos that house mathematics, science, and language skills must be broken down to better replicate how those disciplines mix in the real world.

Combining mathematics with writing promotes students’ ability to analyze, compare facts, and synthesize information.

Written explanations enable teachers to better evaluate students’ mathematical thinking.
PL January 2007

Pink (2005) points out that the knowledge-based society of the United States is now morphing into a conceptual society. One of his key points is that academic knowledge alone will not protect workers from obsolescence and the impact of global competition. For educators, this economic evolution creates even greater urgency for integrated instruction and curriculum. Typically, teachers focusing on integrating curriculum gravitate toward those disciplines they consider kindred: English with social studies, mathematics with science. But this approach is far too limited to prepare students for a truly conceptual age.

A major cause of this societal change is the ease of communicating knowledge and ideas over the Internet. Among the concepts that Pink (2005) indicates will be needed in the future is the ability to turn data and information into a narrative. The direct consequence is that language skills, mathematics, and science learning must be intertwined.

Why Write in Mathematics?
Numerous mathematics educators have cited the benefits of linking writing with mathematics, pointing out that teachers who ask their students to write about mathematics are able to:

- Gain insight into their students’ mathematical thinking
- Diagnose their students’ misconceptions
- Assess students’ study habits and attitudes
- Evaluate their own teaching techniques (National Council of Teachers of Mathematics [NCTM], 2000).

Other educators have focused on the importance of improving mathematical vocabulary, organizing ideas, solving problems, and clarifying mathematical concepts, all areas that combine numeracy with literacy. Miller (1991) points out that “improved mastery of mathematics concepts and skills is possible if students are asked to write about their understanding” (p. 520). Further benefits of combining mathematics with writing include promoting the ability to analyze, compare facts, and synthesize information (Kennedy, 1980; Russek, 2006).

Studies by Freeman and Murphy (1992), Johnson (1983), and Sutton and Kruger (2002) indicate that this integration fosters greater student interest and higher student achievement levels in mathematics. Mathematics, after all, is a written language, and mathematicians write about mathematics.

Challenges
Teachers are often unprepared for the daunting task of integrating mathematics and writing. Mathematics teachers usually worry that teaching writing during mathematics class
takes time away from learning mathematics itself. They may also worry about correcting writing and spending time going over students’ papers because they do not feel competent to examine student writing or they feel it is the role of the language arts teacher. Contrarily, language arts teachers spend little time teaching students genres of writing that rely largely on mathematics and are likely to feel ill at ease with mathematical content.

For the principal who sees the benefits of integrating curriculum, this “silo mentality” of subject separation presents a major challenge. Mathematics teachers often ask:

- How much time should be devoted to writing in mathematics?
- What strategies work best to integrate writing and mathematics?
- What are the genres of mathematical writing that students need to know?
- How can I be sure that the writing strategies I use in teaching mathematics are consistent with the writing demands in other subjects?

Mathematics teachers often see the expectation for integrating writing into the curriculum as an extra burden. Rarely can they recall their English teachers explaining how to write statements in algebra or geometry or their mathematics teachers helping them write a comparative essay about Roman numerals and Arabic numerals. Research by Gopen and Smith (1990), however, indicates that teachers can cover “the [mathematics] material” and “incorporate writing assignments...with significant success and without unduly burdensome effort” (p. 18). This research has also been corroborated by Burns (2004), Reeves (2002), and Countryman (1992).

**Strategies for Mathematics Writing**

A starting point for connecting the silos of mathematics and writing is to accept three key principles:

- Numeracy and literacy cannot be separated in the quest for mathematical achievement
- Writing and learning mathematics are a natural team
- Teachers need a wide range of writing strategies that are specifically suited to mathematics.

To support principals, teachers, and students, we researched the connections between writing and mathematics, then developed appropriate writing strategies that make writing and mathematics a unitary subject. Learning to write requires fluency plus organization. Simply stated, a writer must know the language of the subject or topic and the different types of organizational schema (also called genres) that are pertinent to mathematics. Writers must know the related vocabulary (e.g., operations, polynomials, axis, and perpendicular) to build their fluency and organization.

But knowing the vocabulary is only a starting point. To express their knowledge and application of these vocabulary terms, students must also know a variety of writing formats: definition, explanation, narration, and compare and contrast, among others. In mathematics, “developing fluency requires a balance and connection between conceptual understanding and computational proficiency” (NCTM, 2000, p. 35).

The concept of “fluency plus organization” in mathematics can be understood through the writings of Gardner (1993) who, in his work on multiple intelligences, defines intelligence as “the ability to solve problems or fashion products that are of consequence in a particular cultural setting or community” (p. 15). Gardner speaks of intelligences (plural) rather than intelligence (singular) to denote problem-solving ability in specific domains, such as verbal/linguistic, musical/ rhythmic, and logical/mathematical. He stresses the importance of focusing on both “the content of instruction and the means or medium for communicating that content” (p. 32).

To combine fluency (vocabulary and sentence composing) with organization (writing genres) we have focused on 10 strategies that guide students in writing for mathematics and have created a model called “the Planning Wheel” (figure 1). In this model, mathematics is in the center surrounded by instructional strategies that empower the students to express their learning and knowledge through writing. All of strategies in the Planning Wheel specifically target NCTM (2000) standards.
Personifications and Interactions

Zero D. Cipher, the director of Indian/Arabic Mathematics, writes to Portia B. Romano, the director of Numbers in Rome, in response to her interest in the concept of zero. Such a letter as this can be used as an introduction to algebra. Providing the essential background knowledge about a concept before actually teaching it makes students confident and curious about what is to come.

Dear Dr. Romano:

Thank you for your letter dated I/X/MC in which you ask for a more effective way to have Roman students do the mathematical operations of addition, subtraction, multiplication, and division. Yes, you are right. Your Roman numerals are great for putting on churches and other important edifices, but they have serious limitations when it comes to what we Indians and Arabs call “place value.” In your system, you have to do the computations mentally. On the other hand, we have figured out a regrouping system that uses zero and permits us to compute on paper and with relative ease.

Here’s a quick explanation of how this number we call zero works. For example, when zero is placed to the right of the numeral one or 1 it represents “no ones,” but signifies that the 1 is ten or in the tens place. That 10 is equivalent to your letter X. Now if we place two zeros to the right of the one as in 100, we now have ten of these tens or 10 of these 10s. From 100 or ten tens, we can go to 1000 or one hundred tens. So as you can see, with two symbols we can begin to express an infinite number of numbers. Of course, the only way you can understand infinity is to understand zero, but that is for another letter.

In your system, your students have to write these numbers as: X, C, M. This may be fine for buildings, but as you can see, there is no way that one can compute with your system.

I know this explanation must seem very complicated, but we will be delighted to send our finest mathematicians to Rome. They will get your students started on what we know will be a revolutionary change in the world of computation and a new branch of mathematics. We call this new branch algebra from an Arabic term ilm aljebra, which means to “repair or make better.”

Please do not hesitate to call on us, and in the meantime, we are including gratis our most recent books on place value and the power of zero.

Very sincerely,

Zero D. Cipher

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that focus on the importance of “communicating...mathematical thinking coherently and clearly “ (p. 59).

These writing strategies help students apply their knowledge, gain proficiency in solving word problems, state reasons with proof, make connections between two mathematical ideas or concepts, explain their mathematical representations, and develop an interest in the big picture of mathematics and mathematical history. Further, they provide the springboard for complete curricular integration.

For example, an assignment that uses the Personifications and Interactions strategy requires two students to work together, with each one “personifying” a mathematical idea or concept. Each student writes a letter as this persona, giving explicit mathematical information to a related mathematical persona, including his or her “address” and “story” or narrative (figure 2). These letters carry within them a touch of humor which not only relieves tension and anxiety (and makes us laugh), but “represents one of the highest forms of human intelligence” (Pink, 2005, p. 190).

Breaking Down the Silos
The Personifications and Interactions example is only one of many ways that writing and mathematics can be integrated. It also corresponds to another one of the emerging demands of the conceptual age that Pink calls play (2005).

We know that humor is a powerful learning tool and this type of strategy brings whimsy to what might otherwise be viewed as a dull topic. By extension, integrating writing and mathematics increases student engagement while building student knowledge. Further, when the writing strategies are specifically aligned to word problems, mastering algorithms, or applying mathematical concepts, teachers naturally start integrating curriculum, with the added benefit of making mathematical literacy an enjoyable learning experience. PL

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